

ITEM AND SCORING SAMPLER
2014

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## INTRODUCTION

The Pennsylvania Department of Education (PDE) provides districts and schools with tools to assist in delivering focused instructional programs aligned to the Pennsylvania Core Standards. These tools include the standards, assessment anchor documents, assessment handbooks, and content-based item and scoring samplers. This 2014 Algebra I Item and Scoring Sampler is a useful tool for Pennsylvania educators in preparing students for the Keystone Exams.

This Item and Scoring Sampler contains released operational multiple-choice and constructed-response items that have appeared on previously administered Keystone Exams. These items will not appear on any future Keystone Exams. Released items provide an idea of the types of items that have appeared on operational exams and that will appear on future operational Keystone Exams, and each item has been through a rigorous review process to ensure alignment with the Assessment Anchors and Eligible Content. This sampler includes items that measure a variety of Assessment Anchor or Eligible Content statements, but it does not include sample items for all Assessment Anchor or Eligible Content statements.

The items in this sampler may be used as examples for creating assessment items at the classroom level, and they may also be copied and used as part of a local instructional program. ${ }^{1}$ Classroom teachers may find it beneficial to have students respond to the constructed-response items in this sampler. Educators can then use the sampler as a guide to score the responses either independently or together with colleagues.

## ABOUT THE KEYSTONE EXAMS

The Keystone Exams are end-of-course assessments currently designed to assess proficiencies in Algebra I, Biology, and Literature. The Pennsylvania Department of Education continues to evaluate the implementation schedule for additional subjects, including English Composition, Civics and Government, U.S. History, World History, Algebra II, Geometry, and Chemistry. The Keystone Exams are just one component of Pennsylvania's high school graduation requirements. Students must also earn state-specified credits, fulfill the state's service-learning and attendance requirements, and complete any additional district requirements to receive a Pennsylvania high school diploma.

For detailed information about how the Keystone Exams are being integrated into the Pennsylvania graduation requirements, please contact the Pennsylvania Department of Education or visit the PDE Web site at http://www.education.state.pa.us. Click on the green check mark and select "Keystone Exams."

## Alignment

The Algebra I Keystone Exam consists of exam questions grouped into two modules: Operations and Linear Equations \& Inequalities and Linear Functions and Data Organizations. Each module corresponds to specific content aligned to statements and specifications included in the course-specific assessment anchor documents. The Algebra I content included in the Keystone Algebra I multiple-choice items will align with the Assessment Anchors as defined by the Eligible Content statements. The process skills, directives, and action statements will also specifically align with the Assessment Anchors as defined by the Eligible Content statements.

[^0]The content included in Algebra I constructed-response items aligns with content included in the Eligible Content statements. The process skills, directives, and action statements included in the performance demands of the Algebra I constructed-response items align with specifications included in the Assessment Anchor statements, the Anchor Descriptor statements, and/or the Eligible Content statements. In other words, the verbs or action statements used in the constructed-response items or stems can come from the Eligible Content, Anchor Descriptor, or Assessment Anchor statements.

## Depth of Knowledge

Webb's Depth of Knowledge (DOK) was created by Dr. Norman Webb of the Wisconsin Center for Education Research. Webb's definition of depth of knowledge is the cognitive expectation demanded by standards, curricular activities, and assessment tasks. Webb's DOK includes four levels, from the lowest (basic recall) level to the highest (extended thinking) level.

| Depth of Knowledge |  |
| :--- | :--- |
| Level 1 | Recall |
| Level 2 | Basic Application of Skill/Concept |
| Level 3 | Strategic Thinking |
| Level 4 | Extended Thinking |

Each Keystone item has been through a rigorous review process to ensure that it is as demanding cognitively as what is required by the assigned Assessment Anchor as defined by the Eligible Content. For additional information about depth of knowledge, please visit the PDE Web Site at http://static.pdesas.org/Content/Documents/Keystone_Exams_ Understanding_Depth_of_Knowledge_and_Cognitive_Complexity.pdf.

## Exam Format

The Keystone Exams are delivered in a paper-and-pencil format as well as in a computer-based online format. The multiple-choice items require students to select the best answer from four possible answer options and record their answers in the spaces provided. The correct answer for each multiple-choice item is worth one point. The constructedresponse items require students to develop and write (or construct) their responses. Constructed-response items in Algebra I are scored using item-specific scoring guidelines based on a 0-4-point scale. Each multiple-choice item is designed to take about one to one and a half minutes to complete. Each constructed-response item is designed to take about 10 minutes to complete. The estimated time to respond to a test question is the same for both test formats. During an actual exam administration, students are given additional time as necessary to complete the exam.

## ITEM AND SCORING SAMPLER FORMAT

This sampler includes the test directions, scoring guidelines, and formula sheet that appear in the Keystone Exams. Each sample multiple-choice item is followed by a table that includes the alignment, answer key, DOK, the percentage ${ }^{2}$ of students who chose each answer option, and a brief answer option analysis or rationale. Each constructed-response item is followed by a table that includes the item alignment, DOK, and the mean student score. Additionally, each of the included item-specific scoring guidelines is combined with sample student responses representing each score point to form a practical, item-specific scoring guide. The General Description of Scoring Guidelines for Algebra I used to develop the item-specific scoring guidelines should be used if any additional itemspecific scoring guidelines are created for use within local instructional programs.

Example Multiple-Choice Item Information Table

| Item Information |  |  |
| :--- | :--- | :--- |
| Alignment |  |  | | Assigned |
| :--- |
| AAEC |$\quad$ Option Annotations

Example Constructed-Response Item Information Table

| Alignment | Assigned AAEC | Depth of Knowledge | Assigned DOK | Mean Score |  |
| :--- | :--- | :--- | :--- | :--- | :--- |

[^1]
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## ALGEBRA I EXAM DIRECTIONS

Below are the exam directions available to students in their test booklets. These directions may be used to help students navigate through the exam.

Formulas that you may need to solve questions in this module are found on page 7 of this test booklet. You may refer to the formula page at any time during the exam.

You may use a calculator on this module. When performing operations with $\pi$ (pi), you may use either calculator $\pi$ or the number 3.14.

There are two types of questions in each module.

## Multiple-Choice Questions

These questions will ask you to select an answer from among four choices.

- First read the question and solve the problem on scratch paper. Then choose the correct answer.
- Only one of the answers provided is correct.
- If none of the choices matches your answer, go back and check your work for possible errors.
- Record your answer in the Algebra I answer booklet.


## Constructed-Response Questions

These questions will require you to write your response.

- These questions have more than one part. Be sure to read the directions carefully.
- You cannot receive the highest score for a constructed-response question without completing all the tasks in the question.
- If the question asks you to show your work or explain your reasoning, be sure to show your work or explain your reasoning. However, not all questions will require that you show your work or explain your reasoning. If the question does not require that you show your work or explain your reasoning, you may use the space provided for your work or reasoning, but the work or reasoning will not be scored.
- All responses must be written in the appropriate location within the response box in the Algebra I answer booklet. Some answers may require graphing, plotting, labeling, drawing, or shading. If you use scratch paper to write your draft, be sure to transfer your final response to the Algebra I answer booklet.

If you finish early, you may check your work in Module 1 [or Module 2] only.

- Do not look ahead at the questions in Module 2 [or back at the questions in Module 1] of your exam materials.
- After you have checked your work, close your exam materials.

You may refer to this page at any time during this portion of the exam.

## GENERAL DESCRIPTION OF SCORING GUIDELINES FOR ALGEBRA I

## 4 POINTS

- The response demonstrates a thorough understanding of the mathematical concepts and procedures required by the task.
- The response provides correct answer(s) with clear and complete mathematical procedures shown and a correct explanation, as required by the task. Response may contain a minor "blemish" or omission in work or explanation that does not detract from demonstrating a thorough understanding.


## 3 POINTS

- The response demonstrates a general understanding of the mathematical concepts and procedures required by the task.
- The response and explanation (as required by the task) are mostly complete and correct. The response may have minor errors or omissions that do not detract from demonstrating a general understanding.


## 2 POINTS

- The response demonstrates a partial understanding of the mathematical concepts and procedures required by the task.
- The response is somewhat correct with partial understanding of the required mathematical concepts and/ or procedures demonstrated and/or explained. The response may contain some work that is incomplete or unclear.


## 1 POINT

- The response demonstrates a minimal understanding of the mathematical concepts and procedures required by the task.


## O POINTS

- The response has no correct answer and insufficient evidence to demonstrate any understanding of the mathematical concepts and procedures required by the task for that grade level.


## FORMULA SHEET

Formulas that you may need to work questions in this sampler are found below. You may refer to this page at any time during this module.

You may use calculator $\pi$ or the number 3.14.


## Linear Equations

Slope: $\quad m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$
Point-Slope Formula: $\quad\left(y-y_{1}\right)=m\left(x-x_{1}\right)$

Slope-Intercept Formula: $\quad y=m x+b$

Standard Equation of a Line: $\quad A x+B y=C$

Arithmetic Properties

Additive Inverse: $\quad a+(-a)=0$

Multiplicative Inverse: $\quad a \cdot \frac{1}{a}=1$

Commutative Property: $\quad a+b=b+a$

$$
a \cdot b=b \cdot a
$$

Associative Property: $\quad(a+b)+c=a+(b+c)$ $(a \cdot b) \cdot c=a \cdot(b \cdot c)$

Identity Property: $\quad a+0=a$

$$
a \cdot 1=a
$$

Distributive Property: $\quad a \cdot(b+c)=a \cdot b+a \cdot c$

Multiplicative Property of Zero: $\quad a \cdot 0=0$

Additive Property of Equality:
If $a=b$, then $a+c=b+c$

Multiplicative Property of Equality:
If $a=b$, then $a \cdot c=b \cdot c$

## ALGEBRA I MODULE 1 <br> MULTIPLE-CHOICE ITEMS

1. When factored completely, which is a factor of $12 a x^{2}-3 a$ ?
A. $12 a$
B. $\left(4 x^{2}+1\right)$
C. $3 a$
D. $(4 x-1)$

| Item Information |  |  |  | Option Annotations |
| :---: | :---: | :---: | :---: | :---: |
| Alignment |  | A1.1.1.5.2 |  | A student could determine the correct answer, option C, by factoring $3 a$ from both terms as $3 a\left(4 x^{2}-1\right)$, then factoring the difference of the squares $\left(4 x^{2}-1\right)$ as $(2 x+1)(2 x-1)$. This results in a complete factored expression of $3 a(2 x+1)(2 x-1)$. Of the three possible factors, only $3 a$ is given as an answer choice. <br> A student could arrive at an incorrect answer by factoring incorrectly or by making a sign error. For example, a student could arrive at option D either by thinking the $x$ is factored out with the $3 a$ or by incorrectly factoring $\left(4 x^{2}-1\right)$ as $(4 x+1)(4 x-1)$. |
| Answer Key |  | C |  |  |
| Depth of Knowledge |  | 2 |  |  |
| $p$-values |  |  |  |  |
| A | B | C | D |  |
| 8\% | 38\% | 34\% | 19\% |  |

2. Simplify:

$$
\frac{(x-7)^{2}}{x(x-4)-21} ; x \neq-3,7
$$

A. -14
B. $\frac{7 x+7}{2 x-3}$
C. $\frac{1}{x+3}$
D. $\frac{x-7}{x+3}$

| Item Information |  |  |  | Option Annotations |
| :---: | :---: | :---: | :---: | :---: |
| Alignment |  |  | A1.1.1.5.3 | A student could determine the correct answer, option D, by expanding the denominator as $\frac{(x-7)^{2}}{x^{2}-4 x-21}$, then factoring the numerator and denominator as $\frac{(x-7)(x-7)}{(x-7)(x+3)}$, and then simplifying the expression to $\frac{(x-7)}{(x+3)}$. |
| Answer Key |  |  | D |  |
| Depth of Knowledge |  |  | 2 |  |
| $p$-values |  |  |  |  |
| A | B | C | D |  |
| 13\% | 27\% | 22\% | 36\% | A student could arrive at an incorrect answer by incorrectly canceling variables, terms, or factors. For example, a student could arrive at option C by canceling both $(x-7)$ terms in the numerator along with the one $(x-7)$ term in the denominator. |
|  |  |  |  |  |

3. A person's hair is 8 centimeters long. The equation below can be used to estimate the length $(L)$, in centimeters (cm), that the person's hair will be after $w$ weeks.

$$
L=\frac{w}{4}+8
$$

Based on the equation, what will be the estimated length of the person's hair after 10 weeks?
A. 4.5 cm
B. 8 cm
C. 10 cm
D. 10.5 cm

| Item Information |  |  |  | Option Annotations |
| :---: | :---: | :---: | :---: | :---: |
| Alignment |  |  | A1.1.2.1.1 | A student could determine the correct answer, option D, by substituting 10 for $w$, then simplifying $L=\frac{10}{4}+8=2.5+8=10.5 .$ <br> A student could arrive at an incorrect answer by using the 10 and/or the 8 incorrectly. For example, a student could arrive at option B by substituting 10 for $L$ and then solving for $w: 10=\frac{w}{4}+8$, which becomes $2=\frac{w}{4}$, which becomes $8=w$. |
| Answer Key |  |  | D |  |
| Depth of Knowledge |  |  | 1 |  |
| $p$-values |  |  |  |  |
| A | B | C | D |  |
| 4\% | 6\% | 8\% | 82\% |  |
|  |  |  |  |  |

4. Ms. Bernard monitored the growth of a fish. The fish originally weighed 27 ounces. The fish grew at a rate of 5 ounces per month. The equation below can be used to describe the weight, in ounces, of the fish.

$$
72=27+5 x
$$

Ms. Bernard correctly determined that $x=9$. What does the solution of the equation mean?
A. The fish grew at a rate of 9 ounces per month for 72 months.
B. The fish grew at a rate of 72 ounces per month for 9 months.
C. It took 9 months for the fish to grow to a weight of 72 ounces.
D. It took 72 months for the fish to grow to a weight of 9 ounces.

| tem Information |  |  |  | Option Annotations |
| :---: | :---: | :---: | :---: | :---: |
| Alignment |  | A1.1.2.1.3 |  | A student could determine the correct answer, option C, by interpreting the 72 as the weight, in ounces, of the fish; the 27 as the original weight, in ounces, of the fish; and the 5 as the rate, in ounces per month, the fish grows. When the rate (ounces per month) is multiplied by a number, that number needs to represent the number of months in order for the $5 x$ term to represent a weight, which can then be added to the initial weight (27 ounces) to derive the final weight (72 ounces). <br> A student could arrive at an incorrect answer by incorrectly interpreting what the numbers in the equation represent. For example, a student could arrive at option A by thinking the solution represents the rate, in ounces per month, and the 72 represents the total number of months. |
| Answer Key |  | C |  |  |
| Depth of Knowledge |  | 2 |  |  |
| $p$-values |  |  |  |  |
| A | B | C | D |  |
| 9\% | 7\% | 80\% | 3\% |  |
|  |  |  |  |  |

5. A system of equations is shown below.

$$
\begin{aligned}
& 2 x+2 y=10 \\
& 5 x-2 y=4
\end{aligned}
$$

What is the solution of the system of equations?
A. $(-2,-7)$
B. $(2,7)$
C. $(2,3)$
D. $(3,2)$

| Item Information |  |  |  | Option Annotations |
| :---: | :---: | :---: | :---: | :---: |
| Alignment |  | A1.1.2.2.1 |  | A student could determine the correct answer, option C, by using the elimination method. Adding the first equation to the second yields $7 x=14$. Dividing both sides of the equation by 7 yields $x=2$. Substituting 2 for $x$ in the equation $2 x+2 y=10$ yields $2(2)+2 y=10$. Subtracting 4 from both sides of the equation yields $2 y=6$. Dividing both sides of the equation by 2 yields $y=3$. Written as an ordered pair, the solution is $(2,3)$. |
| Answer Key |  | C |  |  |
| Depth of Knowledge |  | 1 |  |  |
| $p$-values |  |  |  |  |
| A | B | C | D |  |
| 7\% | 13\% | 73\% | 7\% | A student could arrive at an incorrect answer by subtracting the second equation from the first equation or by reversing the values of $x$ and $y$ in the final ordered pair. For example, a student could arrive at option $B$ by subtracting the second equation from the first, resulting in $3 x=6$, which yields $x=2$. Substituting 2 for $x$ in the first equation yields $2(2)+2 y=10$. This equation can then be solved incorrectly for $y$ by adding 4 to 10 , yielding $2 y=14$, which can be simplified to $y=14$. Written as an ordered pair, the incorrect solution is $(2,7)$. |
|  |  |  |  |  |

6. Juan answered all 50 questions on a test. He earned 3 points for each question he answered correctly. He lost 1 point for each question he answered incorrectly. His final test score was 102 points. The system of equations below describes the relationship between the number of questions he answered correctly $(x)$ and the number of questions he answered incorrectly $(y)$.

$$
\begin{aligned}
& x+y=50 \\
& 3 x-y=102
\end{aligned}
$$

Part of the solution of the system of equations is $x=38$. What does this value represent?
A. the number of questions Juan answered correctly
B. the number of questions Juan answered incorrectly
C. the number of points Juan lost from questions he answered incorrectly
D. the number of points Juan earned from questions he answered correctly

| Item Information |  |  |  | Option Annotations |
| :---: | :---: | :---: | :---: | :---: |
| Alignment |  | A1.1.2.2.2 |  | A student could determine the correct answer, option A, by interpreting that the variable $x$ represents the number of questions Juan answers correctly. <br> A student could arrive at an incorrect answer by incorrectly interpreting the meaning of the variables $x$ and $y$. For example, a student could arrive at option D by thinking the variable $x$ represents the number of points Juan earns. |
| Answer Key |  | A |  |  |
| Depth of Knowledge |  | 2 |  |  |
| $p$-values |  |  |  |  |
| A | B | C | D |  |
| 75\% | 6\% | 9\% | 10\% |  |

7. Jason decided that he will sell his stocks if their value per share $(x)$ goes below $\$ 5$ or above $\$ 15$. Which compound inequality represents the values at which Jason will sell his stocks?
A. $x>\$ 5$ or $x<\$ 15$
B. $x<\$ 5$ or $x>\$ 15$
C. $x>\$ 5$ and $x<\$ 15$
D. $x<\$ 5$ and $x>\$ 15$

| Item Information |  |  |  | Option Annotations |
| :---: | :---: | :---: | :---: | :---: |
| Alignment |  | A1.1.3.1.1 |  | A student could determine the correct answer, option B, by interpreting "below $\$ 5$ " as $x<\$ 5$ and "above $\$ 15$ " as $x>\$ 15$. <br> A student could arrive at an incorrect answer by confusing the direction of the inequality signs or by confusing the use of "or" for "and." For example, a student could arrive at option C using > to represent "below \$5" and < to represent "above \$15," as well as thinking both conditions need to occur ("and") instead of only one of the two conditions. |
| Answer Key |  | B |  |  |
| Depth of Knowledge |  | 2 |  |  |
| $p$-values |  |  |  |  |
| A | B | C | D |  |
| 18\% | 61\% | 9\% | 11\% |  |

8. An inequality is shown below.

$$
4 x+2<2 x+9
$$

Which graph shows the solution of the inequality?
A.

B.

C.

D.


| Item Information |  |  |  | Option Annotations |
| :---: | :---: | :---: | :---: | :---: |
| Alignment |  |  | A1.1.3.1.2 | A student could determine the correct answer, option B, by solving the inequality and then graphing its solution. Subtracting $2 x$ from both sides yields $2 x+2<9$. Subtracting 2 from both sides yields $2 x<7$. Dividing both sides by 2 yields $x<3.5$. |
| Answer Key |  | B |  |  |
| Depth of Knowledge |  | 1 |  |  |
| $p$-values |  |  |  | A student could arrive at an incorrect answer by adding the values together or by adding the coefficients together. For |
| A | B | C | D | example, a student could arrive at option D by not dividing the |
| 11\% | 51\% | 16\% | 22\% | sides by 2 , which leaves $2 x<7$, and graphing an open circle at 7 . |

9. A ticket to a baseball game costs $\$ 20$. Each soda sold at the game costs $\$ 5$. Shawn may spend no more than $\$ 50$. He buys 1 ticket and $x$ sodas. Shawn represents this situation with the inequality below.

$$
5 x+20 \leq 50
$$

The solution of the inequality is $x \leq 6$. Which statement best describes the solution of the inequality?
A. Shawn buys 6 or fewer sodas.
B. Shawn buys 6 or fewer tickets.
C. Shawn buys 1 ticket and 5 sodas.
D. Shawn has less than or equal to $\$ 6$ remaining when he leaves the game.

| Item Information |  |  |  | Option Annotations |
| :---: | :---: | :---: | :---: | :---: |
| Alignment |  |  | A1.1.3.1.3 | A student could determine the correct answer, option A, by interpreting the variable $x$ as the number of sodas Shawn can buy and spend no more than $\$ 50$. |
| Answer Key |  |  | A |  |
| Depth of Knowledge |  |  | 2 |  |
| $p$-values |  |  |  | A student could arrive at an incorrect answer by incorrectly interpreting the meaning of the variable $x$ or by incorrectly interpreting what the inequality represents. For example, a student could arrive at option C by seeing that Shawn buying 1 ticket and 5 sodas is a possible solution; however, the inequality $x \leq 6$ represents all possible solutions and not just one possible solution. |
| A | B | C | D |  |
| 71\% | 5\% | 12\% | 11\% |  |
|  |  |  |  |  |

10. A group of friends will buy at most 8 snacks at a movie theater and spend no more than $\$ 42$. They will pay $\$ 4$ for each box of candy and $\$ 7$ for each bag of popcorn. The system of inequalities graphed below represents this information.

Movie Theater Snacks


Which combination of boxes of candy and bags of popcorn could the group buy?
A. 2 boxes of candy and 6 bags of popcorn
B. 3 boxes of candy and 4 bags of popcorn
C. 5 boxes of candy and 4 bags of popcorn
D. 8 boxes of candy and 1 bag of popcorn

| Item Information |  |  |  | Option Annotations |
| :---: | :---: | :---: | :---: | :---: |
| Alignment |  |  | A1.1.3.2.2 | A student could determine the correct answer, option B, by interpreting the combination as the ordered pair $(3,4)$ and identifying the ordered pair as a point within the solution set. <br> A student could arrive at an incorrect answer by incorrectly interpreting the meanings of the variable(s) or by not identifying the ordered pair as a point in the solution set. For example, a student could arrive at option D by identifying the point $(8,1)$ as a solution to the "no more than $\$ 42$ " inequality but failing to notice the point is not part of the solution for the "at most 8 snacks" inequality. |
| Answer Key |  |  | B |  |
| Depth of Knowledge |  |  | 2 |  |
| $p$-values |  |  |  |  |
| A | B | C | D |  |
| 11\% | 64\% | 13\% | 11\% |  |
|  |  |  |  |  |

## CONSTRUCTED-RESPONSE ITEMS

11. A large washtub already contains 6 gallons of water. A faucet is turned on and continues to fill the washtub at a rate of $\frac{1}{2}$ gallon per minute.
A. How many total gallons of water will be in the washtub when the faucet has been on for 5 minutes?
$\qquad$ gallons

When the faucet has been on for $x$ minutes, there will be $y$ gallons of water in the washtub.
B. Write a linear equation to model the number of gallons of water $(y)$ in the washtub $x$ minutes after the faucet has been turned on.
linear equation: $\qquad$
C. Using your equation, determine the number of minutes from when the faucet is turned on until there are exactly $23 \frac{3}{4}$ gallons of water in the washtub.
$\qquad$ minutes

## Go to the next page to finish question 11.

11. Continued. Please refer to the previous page for task explanation.

A second washtub already contains 2 gallons of water. A larger faucet is used to fill this washtub at a rate $1 \frac{1}{2}$ times the rate of the first faucet.

Both faucets are turned on at the same time.
D. Determine the number of minutes until both washtubs contain the same number of gallons of water.
$\qquad$ minutes

## SCORING GUIDE

## \#11 ITEM INFORMATION

| Alignment | A1.1.2 | Depth of Knowledge | 3 | Mean Score | 1.52 |
| :--- | :--- | :--- | :--- | :--- | :--- |

## ITEM-SPECIFIC SCORING GUIDELINE

| Score | Description |
| :---: | :--- |
| $\mathbf{4}$ | The student demonstrates a thorough understanding of writing, solving, and/or applying a <br> linear equation and interpreting solutions to problems in the context of the problem situation <br> (linear equations only) by correctly solving problems and clearly explaining procedures. |
| $\mathbf{3}$ | The student demonstrates a general understanding of writing, solving, and/or applying a linear <br> equation and interpreting solutions to problems in the context of the problem situation (linear <br> equations only) by correctly solving problems and clearly explaining procedures with only <br> minor errors or omissions. |
| $\mathbf{2}$ | The student demonstrates a partial understanding of writing, solving, and/or applying a linear <br> equation and interpreting solutions to problems in the context of the problem situation (linear <br> equations only) by correctly performing a significant portion of the required task. |
| $\mathbf{1}$ | The student demonstrates minimal understanding of writing, solving, and/or applying a linear <br> equation and interpreting solutions to problems in the context of the problem situation (linear <br> equations only). |
| $\mathbf{0}$ | The student does not demonstrate any understanding of the mathematical concepts and <br> procedures as required by the task. Response may show only information copied from the <br> question. |

## Top Scoring Response:

| Part A Answer |  |
| :--- | :--- |
| What? |  |
| $8 \frac{1}{2}$ |  |
| OR equivalent |  |

(1 score point)
1 point for correct answer

| Part B Answer |  |
| :---: | :---: |
| What? |  |
| $y=\frac{1}{2} x+6$ |  |
| OR equivalent |  |

(1 score point)
1 point for correct answer

| Part C Answer |  |
| :---: | :---: |
| What? |  |
| $35 \frac{1}{2}$ |  |
| OR equivalent |  |

## (1 score point)

1 point for correct answer

| Part D Answer |  |
| :--- | :--- |
| What? |  |
| 16 |  |

## (1 score point)

1 point for correct answer

## STUDENT RESPONSE

## RESPONSE SCORE: 4 POINTS

11. A large washtub already contains 6 gallons of water. A faucet is turned on and continues to fill the washtub at a rate of $\frac{1}{2}$ gallon per minute.
A. How many total gallons of water will be in the washtub when the faucet has been on for 5 minutes?

Student has given a correct answer, $8 \frac{1}{2}$.
$81 / 2$
gallons

When the faucet has been on for $x$ minutes, there will be $y$ gallons of water in the washtub.
B. Write a linear equation to model the number of gallons of water $(y)$ in the washtub $x$ minutes after the faucet has been turned on.

Student has given a correct equation, $1 \_2 x+6=y$.
C. Using your equation, determine the number of minutes from when the faucet is turned on until there are exactly $23 \frac{3}{4}$ gallons of water in the washtub.
 minutes

Student has given a correct answer, $35 \frac{1}{2}$.

## Go to the next page to finish question 11.

11. Continued. Please refer to the previous page for task explanation.

A second washtub already contains 2 gallons of water. A larger faucet is used to fill this washtub at a rate $1 \frac{1}{2}$ times the rate of the first faucet.

Both faucets are turned on at the same time.
D. Determine the number of minutes until both washtubs contain the same number of gallons of water.

$$
\begin{array}{ll}
3 / 4 x+2=y & 3 / 4(16)+2=14 \\
1 / 2 x+6=4 & 1 / 2(16)+6=14
\end{array}
$$

## STUDENT RESPONSE

## RESPONSE SCORE: 3 POINTS

11. A large washtub already contains 6 gallons of water. A faucet is turned on and continues to fill the washtub at a rate of $\frac{1}{2}$ gallon per minute.
A. How many total gallons of water will be in the washtub when the faucet has been on for 5 minutes?

Student has given a correct answer, 8.5.

When the faucet has been on for $x$ minutes, there will be $y$ gallons of water in the washtub.
B. Write a linear equation to model the number of gallons of water $(y)$ in the washtub $x$ minutes after the faucet has been turned on.
linear equation: $6+y=\frac{1}{2} x$

Student has given an incorrect equation, $6+y=\frac{1}{2} x$. Manipulating this equation algebraically does not produce a correct equation.
C. Using your equation, determine the number of minutes from when the faucet is turned on until there are exactly $23 \frac{3}{4}$ gallons of water in the washtub.

$$
35.5
$$

minutes

Student has given a correct answer, 35.5, even though an incorrect equation was given in part $B$.

## Go to the next page to finish question 11.

11. Continued. Please refer to the previous page for task explanation.

A second washtub already contains 2 gallons of water. A larger faucet is used to fill this washtub at a rate $1 \frac{1}{2}$ times the rate of the first faucet.

Both faucets are turned on at the same time.
D. Determine the number of minutes until both washtubs contain the same number of gallons of water.
$\frac{3}{4} 10 \mathrm{~min}=7.5 \mathrm{gal}+2=9.5 \mathrm{gal}$
$\frac{1}{2} 10 \mathrm{~min}=5 \mathrm{gal}+6=11 \mathrm{gal}$
$\frac{3}{4} 15 \mathrm{~min}=11.25 \mathrm{gal}+2=13.25 \mathrm{gal}$
$\frac{1}{2} \quad 15 \mathrm{~min}=7.5 \mathrm{gal}+6=13.5 \mathrm{gal}$
$\frac{3}{4} 16 \mathrm{~min}=12 \mathrm{gal}+2=14 \mathrm{gal}$
$\frac{1}{2} 16$ min $-8 \mathrm{gal}+6=14 \mathrm{gal}$

Student has given a correct answer, 16.
$\qquad$ minutes

## STUDENT RESPONSE

RESPONSE SCORE: 2 POINTS



## STUDENT RESPONSE

## RESPONSE SCORE: 1 POINT

11. A large washtub already contains 6 gallons of water. A faucet is turned on and continues to fill the washtub at a rate of $\frac{1}{2}$ gallon per minute.
A. How many total gallons of water will be in the washtub when the faucet has been on for 5 minutes?
$81 / 2$ gallons

Student has given a correct answer, $8 \frac{1}{2}$.

When the faucet has been on for $x$ minutes, there will be $y$ gallons of water in the washtub.
B. Write a linear equation to model the number of gallons of water $(y)$ in the washtub $x$ minutes after the faucet has been turned on.

linear equation:


Student has given an incorrect equation, $y=y z x+n$.
C. Using your equation, determine the number of minutes from when the faucet is turned on until there are exactly $23 \frac{3}{4}$ gallons of water in the washtub.

Student has given an incorrect answer, $47 \frac{1}{2}$.

## Go to the next page to finish question 11.

11. Continued. Please refer to the previous page for task explanation.

A second washtub already contains 2 gallons of water. A larger faucet is used to fill this washtub at a rate $1 \frac{1}{2}$ times the rate of the first faucet.

Both faucets are turned on at the same time.
D. Determine the number of minutes until both washtubs contain the same number of gallons of water.

$2 \quad 2 \frac{3}{4} \quad 3 \frac{2}{4} \quad 4 \frac{1}{4} \quad 5$
5
$51 / 2$
6
$6 \frac{1}{2}$
7

1
$\begin{array}{lllllll}5 \frac{3}{4} & 6 \frac{2}{4} & 7 \frac{1}{4} & 8 & 8 \frac{3}{4} & 9 \frac{2}{4} & 10 \frac{1}{4} \\ 11\end{array}$
$\begin{array}{llllllll}7 \frac{1}{2} & 8 & 8 \frac{1}{2} & 9 & 9 \frac{1}{2} & 10 & 10 \frac{1}{2} & 11\end{array}$
$\begin{array}{llllllll}5 & 6 & 7 & 8 & 9 & 10 & 11 & 12\end{array}$

Student has given an incorrect answer, 12.
$\qquad$ minutes

## STUDENT RESPONSE

RESPONSE SCORE: 0 POINTS


12. Rolando drives at least 40 miles but less than 60 miles each week.
A. Graph the compound inequality representing all of the possible distances Rolando could drive for 8 weeks.

Rolando's Driving for 8 Weeks

B. Explain why you chose to use the symbols you used for the endpoints of the compound inequality in part A.

Go to the next page to finish question 12.
12. Continued. Please refer to the previous page for task explanation.

Rolando buys at least 8.5 but no more than 11 gallons of gas each week. The price of gas has ranged from $\$ 2.40$ to $\$ 2.65$ per gallon each week.
C. Write an inequality to model all of the possible amounts of money $(m)$ Rolando spends on gas each week. Show or explain all your work.

## SCORING GUIDE

## \#12 ITEM INFORMATION

| Alignment | A1.1.3 | Depth of Knowledge | 3 | Mean Score | 1.14 |
| :--- | :--- | :--- | :--- | :--- | :--- |

## ITEM-SPECIFIC SCORING GUIDELINE

| Score | Description |
| :---: | :--- |
| $\mathbf{4}$ | The student demonstrates a thorough understanding of writing compound inequalities <br> and graphing their solution sets on a number line by correctly solving problems and clearly <br> explaining procedures. |
| $\mathbf{3}$ | The student demonstrates a general understanding of writing compound inequalities and <br> graphing their solution sets on a number line by correctly solving problems and clearly <br> explaining procedures with only minor errors or omissions. |
| $\mathbf{2}$ | The student demonstrates a partial understanding of writing compound inequalities and <br> graphing their solution sets on a number line by correctly performing a significant portion of <br> the required task. |
| $\mathbf{1}$ | The student demonstrates minimal understanding of writing compound inequalities and <br> graphing their solution sets on a number line. |
| $\mathbf{0}$ | The student does not demonstrate any understanding of the mathematical concepts and <br> procedures as required by the task. Response may show only information copied from the <br> question. |

## Top Scoring Response:

| Part A Answer |  |
| :---: | :---: |
| What? | Why? |
| Rolando's Driving for 8 Weeks |  |

(1 score point)
1 point for correct answer
OR $1 / 2$ point for correct endpoints

| Part B Answer |  |
| :---: | :--- |
| What? | Why? |
|  | I chose a solid (closed) circle at 320 since Rolando would drive greater <br> than or equal to $8 \times 40=320$ miles. I chose an open circle at 480 since <br> Rolando would drive less than, but not equal to, $8 \times 60=480$ miles. <br> OR equivalent |

(1 score point)
1 point for complete explanation
OR $1 / 2$ point for correct but incomplete support

| Part C Answer |  |
| :---: | :---: |
| What? | Why? |
| $20.40 \leq m \leq 29.15$ <br> OR equivalent | Minimum: 8.5 gallons $\times \$ 2.40$ per gallon $=\$ 20.40$ <br> Maximum: 11 gallons $\times \$ 2.65$ per gallon $=\$ 29.15$ <br> OR <br> The least amount Rolando could pay for gas is when he buys 8.5 gallons at $\$ 2.40$ per gallon, which is $\$ 20.40$. The greatest amount Rolando could pay for gas is when he buys 11 gallons at $\$ 2.65$ per gallon, which is $\$ 29.15$. Since Rolando could pay either of these amounts, I used the less than or equal to symbols. <br> OR equivalent |

## (2 score points)

1 point for correct answer
OR $1 / 2$ point for partially correct answer (1 calculation error or 1 sign error)
1 point for complete support
OR $1 / 2$ point for correct but incomplete support

## STUDENT RESPONSE

RESPONSE SCORE: 4 POINTS



## STUDENT RESPONSE

RESPONSE SCORE: 3 POINTS



STUDENT RESPONSE
RESPONSE SCORE: 2 POINTS
12. Rolando drives at least 40 miles but less than 60 miles each week.
A. Graph the compound inequality representing all of the possible distances Rolando could drive for 8 weeks.

Rolando's Driving for 8 Weeks

$40 \geq x$
$\times 8$
$60<x$
$\times 8$
$320 \geq x$
Student has correctly graphed the compound inequality.
$480<x$
B. Explain why you chose to use the symbols you used for the endpoints of the compound inequality in part A.
1 chose $3202 x$ becanse at the top it said that Rolanato drives at least 40 miles each week. At least means it is greater than or equal to because he never drives less. Then I multiplied $40 \times 8$ since it was for the time span of 8 weeks and got $320 \geq x$. In the second equation Rolando drives less than 60 miles each week. Less than means that he always drives under 60 miles/week, Then, 1 multiplied 60 by 8 , for the 8 weeks, and got $480<x$.

Student's explanation is incorrect because it explains the inequality symbols, not the endpoints.

Go to the next page to finish question 12.
12. Continued. Please refer to the previous page for task explanation.

Rolando buys at least 8.5 but no more than 11 gallons of gas each week. The price of gas has ranged from $\$ 2.40$ to $\$ 2.65$ per gallon each week.
C. Write an inequality to model all of the possible amounts of money $(m)$ Rolando spends on gas each week. Show or explain all your work.

$8.5 \cdot 2.40$
$11 \cdot 2.65$
$\$ 20.40 \quad \$ 29.15$

$$
20.40 \leq x \geq 29.15
$$



Student has a partially correct inequality (contains one sign error but is otherwise correct) and has shown all work necessary to support the inequality.

## STUDENT RESPONSE

RESPONSE SCORE: 1 POINT



## STUDENT RESPONSE

## RESPONSE SCORE: 0 POINTS

12. Rolando drives at least 40 miles but less than 60 miles each week.
A. Graph the compound inequality representing all of the possible distances Rolando could drive for 8 weeks.

Rolando's Driving for 8 Weeks


Student's graph is incorrect.
B. Explain why you chose to use the symbols you used for the endpoints of the compound inequality in part A.

I put them like that because open circle means equal to the dark circle means les than or greater than.

> Student's explanation is incorrect.

## Go to the next page to finish question 12.

12. Continued. Please refer to the previous page for task explanation.

Rolando buys at least 8.5 but no more than 11 gallons of gas each week. The price of gas has ranged from $\$ 2.40$ to $\$ 2.65$ per gallon each week.
C. Write an inequality to model all of the possible amounts of money $(m)$ Rolando spends on gas each week. Show or explain all your work.

$$
m=8.5 \leq 11=2.40 \leq 2.60
$$

Student's inequality is incorrect and no support is attempted.

## ALGEBRA I MODULE 1—SUMMARY DATA

## MULTIPLE-CHOICE

| Sample <br> Number | Alignment | Answer <br> Key | Depth of <br> Knowledge | $\boldsymbol{p}$-values |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A1.1.1.5.2 |  | 2 | B | C | D |  |  |
| 1 | A1.1.1.5.3 | D | 2 | $8 \%$ | $38 \%$ | $34 \%$ | $19 \%$ |  |
| 2 | A1.1.1.5. | $13 \%$ | $27 \%$ | $22 \%$ | $36 \%$ |  |  |  |
| 3 | A1.1.2.1.1 | D | 1 | $4 \%$ | $6 \%$ | $8 \%$ | $82 \%$ |  |
| 4 | A1.1.2.1.3 | C | 2 | $9 \%$ | $7 \%$ | $80 \%$ | $3 \%$ |  |
| 5 | A1.1.2.2.1 | C | 1 | $7 \%$ | $13 \%$ | $73 \%$ | $7 \%$ |  |
| 6 | A1.1.2.2.2 | A | 2 | $75 \%$ | $6 \%$ | $9 \%$ | $10 \%$ |  |
| 7 | A1.1.3.1.1 | B | 2 | $18 \%$ | $61 \%$ | $9 \%$ | $11 \%$ |  |
| 8 | A1.1.3.1.2 | B | 1 | $11 \%$ | $51 \%$ | $16 \%$ | $22 \%$ |  |
| 9 | A1.1.3.1.3 | A | 2 | $71 \%$ | $5 \%$ | $12 \%$ | $11 \%$ |  |
| 10 | A1.1.3.2.2 | B | 2 | $11 \%$ | $64 \%$ | $13 \%$ | $11 \%$ |  |

## CONSTRUCTED-RESPONSE

| Sample <br> Number | Alignment | Points | Depth of <br> Knowledge | Mean Score |
| :---: | :---: | :---: | :---: | :---: |
| 11 | A1.1.2 | 4 | 3 | 1.52 |
| 12 | A1.1.3 | 4 | 3 | 1.14 |

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## ALGEBRA I MODULE 2

## MULTIPLE-CHOICE ITEMS

1. The set of ordered pairs shown below is a relation that is a function of $x$.

$$
\{(1,3),(2,4),(3,5),(4,6)\}
$$

Which ordered pair could be included in the set so that the relation remains a function of $x$ ?
A. $(0,4)$
B. $(1,6)$
C. $(3,3)$
D. $(4,7)$

| Item Information |  |  |  | Option Annotations |
| :---: | :---: | :---: | :---: | :---: |
| Alignment |  |  | A1.2.1.1.2 | A student could determine the correct answer, option A, by identifying the ordered pair that does not have the same $x$-value as any ordered pair in the relation. <br> A student could arrive at an incorrect answer by using the $y$-value. For example, a student could arrive at option D by identifying the ordered pair that does not have the same $y$-value as any ordered pair in the relation. |
| Answer Key |  |  | A |  |
| Depth of Knowledge |  |  | 2 |  |
| $p$-values |  |  |  |  |
| A | B | C | D |  |
| 38\% | 14\% | 15\% | 32\% |  |

2. Aki wants to buy a music player that costs $\$ 234$ using only the money he earned from mowing lawns. The table below shows the amount of money Aki earned as a function of the number of lawns he mowed.

| Aki's Mowing |
| :---: | :---: |
| Number <br> of Lawns <br> Mowed Amount <br> Earned <br> $\mathbf{( \$ )}$ <br> 3 24 <br> 6 48 <br> 8 64 <br> 11 88 |

Based on the function shown in the table, what is the least number of lawns Aki will have to mow to buy the music player?
A. 22
B. 29
C. 30
D. 31

| Item Information |  |  |  | Option Annotations |
| :---: | :---: | :---: | :---: | :---: |
| Alignment |  |  | A1.2.1.2.1 | A student could determine the correct answer, option C , by using the table of the linear function and creating the equation $y=8 x$. Substituting 234 for $y$ yields $234=8 x$. Dividing both sides by 8 yields $29.25=x$. Since the context (number of lawns) requires a whole number answer that yields a value of $\$ 234$ or more, the least whole number value is 30 . <br> A student could arrive at an incorrect answer by incorrectly interpreting the table or by incorrectly applying the linear function equation. For example, a student could arrive at option B by correctly finding $29.25=x$ but then rounding the number without verifying the expected output-value. |
| Answer Key |  |  | C |  |
| Depth of Knowledge |  | 2 |  |  |
| $p$-values |  |  |  |  |
| A | B | C | D |  |
| 10\% | 18\% | 65\% | 6\% |  |
|  |  |  |  |  |

3. The table below represents a function of $x$.

| $\boldsymbol{x}$ | $\boldsymbol{y}$ |
| ---: | ---: |
| -1 | -13 |
| 2 | -1 |
| 5 | 11 |
| 7 | 19 |

Which equation describes the function?
A. $y=x-12$
B. $y=2 x+5$
C. $y=4 x-9$
D. $y=-4 x+13$

| Item Information |  |  | Option Annotations |
| :---: | :---: | :---: | :---: |
| Alignment |  |  | A1.2.1.2.2 | | A student could determine the correct answer, option C, by |
| :--- |
| identifying the linear equation that will map every value of $x$ in the |
| table to the corresponding value of $y$. |

4. LaShawn earned $\$ 60.00$ for working 8 hours this weekend. What is the total amount of money LaShawn would earn for working 34 hours at the same rate of pay?
A. $\$ 255$
B. $\$ 272$
C. $\$ 315$
D. $\$ 453$

| Item Information |  |  |  | Option Annotations |
| :---: | :---: | :---: | :---: | :---: |
| Alignment |  | A1.2.2.1.1 |  | A student could determine the correct answer, option A, by calculating and using the constant rate of change. Dividing $\$ 60.00$ by 8 hours yields a constant rate of change of $\$ 7.50$ per hour. Multiplying the rate by 34 hours yields $\$ 255$. <br> A student could arrive at an incorrect answer by incorrectly calculating the rate or by applying the rate incorrectly. For example, a student could arrive at option D by dividing 8 by 60, applying this rate to 34 hours, and then moving the decimal until the value is greater than $\$ 60$. |
| Answer Key |  |  | A |  |
| Depth of Knowledge |  |  | 2 |  |
| $p$-values |  |  |  |  |
| A | B | C | D |  |
| 83\% | 8\% | 5\% | 4\% |  |

5. A function of $x$ is graphed on the coordinate plane below.


What is the slope of the graph?
A. 0
B. $\frac{1}{5}$
C. 5
D. undefined

| Item Information |  |  |  | Option Annotations |
| :---: | :---: | :---: | :---: | :---: |
| Alignment |  |  | A1.2.2.1.4 | A student could determine the correct answer, option A, by selecting two points on the graph and applying these points to the formula $m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}$ or by recognizing that the graph of a horizontal line has a slope of 0 . <br> A student could arrive at an incorrect answer by finding the $y$-intercept or by using the reciprocal. For example, a student could arrive at option C by identifying the $y$-intercept of the graph. |
| Answer Key |  |  | A |  |
| Depth of Knowledge |  |  | 2 |  |
| $p$-values |  |  |  |  |
| A | B | C | D |  |
| 53\% | 2\% | 11\% | 34\% |  |
|  |  |  |  |  |

6. Marcie heated a beaker of water in science class. The scatter plot below shows the temperature ( $y$ ), in degrees Celsius $\left({ }^{\circ} \mathrm{C}\right)$, of the water based on the number of minutes $(x)$ she heated the water.

## Temperatures of Water in Beaker



Which equation describes the line of best fit for the temperature of the water based on the number of minutes Marcie heated the water?
A. $y=5.3 x+12$
B. $y=5.3 x+23$
C. $y=-5.3 x+23$
D. $y=-5.3 x+50$

| Item Information |  |  |  | Option Annotations |
| :---: | :---: | :---: | :---: | :---: |
| Alignment |  |  | A1.2.2.2.1 | A student could determine the correct answer, option B, by examining the scatter plot and identifying that the $y$-intercept should be between 20 and 25 . Since the points have a positive correlation, the student could also determine the slope is positive. <br> A student could arrive at an incorrect answer by incorrectly identifying the location of the $y$-intercept or by switching the sign for the slope. For example, a student could arrive at option D by using a negative slope and by incorrectly identifying the $y$-intercept as the grid line that is closest to the greatest $y$-value for the points on the scatter plot. |
| Answer Key |  |  | B |  |
| Depth of Knowledge |  |  | 2 |  |
| $p$-values |  |  |  |  |
| A | B | C | D |  |
| 19\% | 61\% | 11\% | 9\% |  |
|  |  |  |  |  |

7. Javier's score on a science test is equal to the upper quartile value of all the scores on the test. Based on this information, which statement about Javier's score is most likely to be true?
A. Javier's score is 75 .
B. Javier's score is greater than 75 other scores.
C. Javier's score is the same as $75 \%$ of all the scores.
D. Javier's score is greater than $75 \%$ of all the scores.

| Item Information |  |  |  | Option Annotations |
| :---: | :---: | :---: | :---: | :---: |
| Alignment |  |  | A1.2.3.1.1 | A student could determine the correct answer, option D, by interpreting the upper quartile value as being the value for which $75 \%$ of the data has a lower value. <br> A student could arrive at an incorrect answer by incorrectly identifying how the $75 \%$ relates to the data set. For example, a student could arrive at option A by incorrectly interpreting the 75 as the actual score on the test (i.e., Javier earned $75 \%$ of the points on a 100-point test). |
| Answer Key |  |  |  |  |
| Depth of Knowledge |  |  |  |  |
| $p$-values |  |  |  |  |
| A | B | C | D |  |
| 11\% | 10\% | 31\% | 48\% |  |

8. The circle graph below shows the percent of the total number of students enrolled in a high school who are in each grade.

## School Enrollment



There are currently 448 freshmen enrolled in the high school. About 75\% of the seniors enrolled in the high school will attend college next year. Which is most likely the number of seniors currently enrolled in the high school who will attend college next year?
A. 167
B. 288
C. 336
D. 384

| Item Information |  |  |  | Option Annotations |
| :---: | :---: | :---: | :---: | :---: |
| Alignment |  |  | A1.2.3.2.1 | A student could determine the correct answer, option B, by using the number of freshmen to calculate the number of students in the school. Dividing the number of freshmen (448) by the percentage of the students who are freshmen (0.28) yields 1,600 students in the school. Multiplying this total by the percentage of students who are seniors ( 0.24 ) yields 384 seniors. Multiplying the number of seniors by the percentage of seniors who will attend college next year ( 0.75 ) yields 288 seniors who will attend college next year. <br> A student could arrive at an incorrect answer by using incorrect calculations or by not following through to the end of the calculation. For example, a student could arrive at option C by finding $75 \%$ of the number of freshmen. |
| Answer Key |  |  | B |  |
| Depth of Knowledge |  | 2 |  |  |
| $p$-values |  |  |  |  |
| A | B | C | D |  |
| 12\% | 37\% | 37\% | 14\% |  |
|  |  |  |  |  |

9. Four violin students recorded the number of days they practiced violin each month for a year. Which stem-and-leaf plot has mode and median values that are equal?
A. Number of Days Practiced Each Month
B. Number of Days Practiced Each Month

| 0 |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 1 | 1 | 4 |  |  |
| 2 | 1 | 1 | 1 | 1 |  |
| 3 | 1 | 1 | 1 | 1 | 1 |


| 0 | 1 | 5 | 6 | 8 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 0 | 4 | 5 | 8 |  |
| 2 | 2 | 5 |  |  |  |
| 3 | 0 |  |  |  |  |


| Key |
| :---: |
| $1 \mid 2=12$ days |


| Key |
| :---: |
| $1 \mid 2=12$ days |

C. $\begin{gathered}\text { Number of Days } \\ \text { Practiced Each Month }\end{gathered}$

| 0 | 2 | 3 | 4 | 5 | 6 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 3 | 3 |  |  |  |  |
| 2 | 0 | 1 |  |  |  |  |
| 3 | 0 | 1 |  |  |  |  |
|  |  |  |  |  |  |  |


| Key |
| :---: |
| $1 \mid 2=12$ days |

D. Number of Days Practiced Each Month

| 0 | 4 | 9 |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 1 | 1 |  |  |  |  |
| 3 | 0 | 1 |  |  |  |  |


| Key |
| :---: |
| $1 \mid 2=12$ days |


| Item Information |  |  |  |  |
| ---: | :---: | :---: | :---: | :---: |
| Alignment | A1.2.3.2.2 |  |  |  |
| Answer Key |  |  |  | D |
| Depth of Knowledge | 2 |  |  |  |
| p-values |  |  |  |  |
| A | B | C |  |  |
| $20 \%$ | $8 \%$ | $14 \%$ |  |  |
| $58 \%$ |  |  |  |  |


| Option Annotations |
| :--- |
| A student could determine the correct answer, option D, by |
| calculating the mode and median of each stem-and-leaf plot and |
| comparing the values. |
| A student could arrive at an incorrect answer by incorrectly |
| interpreting the stem-and-leaf plots or by incorrectly determining |
| the mode and/or the median. For example, a student could arrive |
| at option A by incorrectly using only the leaf values to determine |
| the mode and median and not applying the stem values. |

10. The scatter plot below shows the arm spans and heights of 20 people in Dorian's class.


Based on the line of best fit, which is most likely the height of a person with an arm span of 200 cm ?
A. 188 cm
B. 192 cm
C. 197 cm
D. 205 cm


## CONSTRUCTED-RESPONSE ITEMS

11. Albert sells baseball programs at a stadium. The function $m(x)=2.50 x$ represents the total amount of money collected, in dollars, for selling $x$ baseball programs.
A. Fill in the table with the amounts of money collected for selling baseball programs.

Albert's Revenue

| Baseball <br> Programs <br> Sold | Money <br> Collected (\$) |
| :---: | :---: |
| 150 |  |
| 175 |  |
| 197 |  |

The cost, in dollars, to print up $x$ programs for each game is represented by the function $c(x)=0.50 x+40$.
B. On the grid below, draw a line that contains the coordinate points of the cost to print up $x$ programs for each game.


Go to the next page to finish question 11.
11. Continued. Please refer to the previous page for task explanation.

In addition to his hourly wage, Albert earns a bonus when the amount of money collected is greater than the cost to print the total number of programs he sold. His bonus is equal to $\frac{1}{2}$ of the difference between the amount of money collected, $m(x)=2.50 x$, and the cost, $c(x)=0.5 x+40$.
C. How much money does Albert earn as a bonus when he sells 309 baseball programs? Show all of your work. Explain why you did each step.

## SCORING GUIDE

## \#11 ITEM INFORMATION

| Alignment | A1.2.1 | Depth of Knowledge | 3 | Mean Score | 1.62 |
| :--- | :--- | :--- | :--- | :--- | :--- |

## ITEM-SPECIFIC SCORING GUIDELINE

| Score | Description |
| :---: | :--- |
| $\mathbf{4}$ | The student demonstrates a thorough understanding of creating, interpreting, and using the <br> equation, graph, or table of a linear function and translating from one representation of a linear <br> function to another (graph, table, and equation) by correctly solving problems and clearly <br> explaining procedures. |
| $\mathbf{3}$ | The student demonstrates a general understanding of creating, interpreting, and using the <br> equation, graph, or table of a linear function and translating from one representation of a linear <br> function to another (graph, table, and equation) by correctly solving problems and clearly <br> explaining procedures with only minor errors or omissions. |
| $\mathbf{2}$ | The student demonstrates a partial understanding of creating, interpreting, and using the <br> equation, graph, or table of a linear function and translating from one representation of a linear <br> function to another (graph, table, and equation) by correctly performing a significant portion of <br> the required task. |
| $\mathbf{1}$ | The student demonstrates minimal understanding of creating, interpreting, and using the <br> equation, graph, or table of a linear function and translating from one representation of a linear <br> function to another (graph, table, and equation). |
| $\mathbf{0}$ | The student does not demonstrate any understanding of the mathematical concepts and <br> procedures as required by the task. Response may show only information copied from the <br> question. |

## Top Scoring Response:

| Part A Answer |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| What? |  |  |  |  |  |  |
|  | Albert's Revenue |  |  |  |  |  |
|  | Baseball <br> Programs <br> Sold | Money <br> Collected (\$) |  |  |  |  |
| 150 | 375.00 |  |  |  |  |  |
|  |  |  |  |  |  |  |
| 175 | 437.50 |  |  |  |  |  |
| 197 | 492.50 |  |  |  |  |  |
|  |  |  |  |  |  |  |

## (1 score point)

1 point for correct answer


## (1 score point)

1 point for correct answer
OR $1 / 2$ point for correct slope only ( 0.50 )
OR $1 / 2$ point for correct $y$-intercept only (40)

| Part C Answer |  |
| :--- | :--- |
| What? | Why? |
| $\$ 289$ | $m(309)=2.50(309)=\$ 772.50$ |
|  | $c(309)=0.50(309)+40=\$ 194.50$ |
|  | $\frac{1}{2}(772.50-194.50)=\frac{1}{2}(578)=\$ 289$. |
|  | AND |
|  | First I substituted 309 into the revenue function and got $\$ 772.50$. Then I substituted 309 into the <br>  <br>  <br>  <br>  <br>  <br> Cost function and got $\$ 194.50$. Then I found the difference $(\$ 578)$, and then found half of that $(\$ 289)$. <br> OR equivalent |

## (2 score points)

1 point for correct answer
1 point for complete support OR $1 / 2$ point for correct but incomplete support

## STUDENT RESPONSE

## RESPONSE SCORE: 4 POINTS

11. Albert sells baseball programs at a stadium. The function $m(x)=2.50 x$ represents the total amount of money collected, in dollars, for selling $x$ baseball programs.
A. Fill in the table with the amounts of money collected for selling baseball programs.

Albert's Revenue

| Baseball <br> Programs <br> Sold | Money <br> Collected (\$) |
| :---: | :--- |
| 150 | $\$ 375$ |
| 175 | $\$ 437.50$ |
| 197 | $\$ 492.50$ |

Student has correctly filled in the table.

The cost, in dollars, to print up $x$ programs for each game is represented by the function $c(x)=0.50 x+40$.
B. On the grid below, draw a line that contains the coordinate points of the cost to print up $x$ programs for each game.


Student has correctly drawn the line on the grid.

Go to the next page to finish question 11.
11. Continued. Please refer to the previous page for task explanation.

In addition to his hourly wage, Albert earns a bonus when the amount of money collected is greater than the cost to print the total number of programs he sold. His bonus is equal to $\frac{1}{2}$ of the difference between the amount of money collected, $m(x)=2.50 x$, and the cost, $c(x)=0.5 x+40$.
C. How much money does Albert earn as a bonus when he sells 309 baseball programs? Show all of your work. Explain why you did each step.

## STEP 1

SUBSTITUTION. I SUBSTITUTE
309 INTO THE $X$ SPOT FOR
RoT EauntTon's AND Solis
TO FIND THE 2 DIFFERENT PRIES
$m(x)=2.5(3009)$
$M(x)=772.50$
$c(x)=.5(309)+40$
$c(x)=194.5$

$\operatorname{step} 2$
SUBTRACTION, I SUBTRACT THE COST to MAKE THE PROGRANS FROM THE PROFIT OF THE PROGRAMS TO HIND THE PROMO.

$$
772.5-194.5=578
$$

$$
\text { STEP } 3
$$

DIVISION, I DIVIDE 528 BY Tho SINGE THE PROBLEM STATES ALBERT EARNS $1 / 2$ DIE DIFFERENCES BETWEEN THE COLECTED MONEY AND PRITE Jo MAKE PROGRAMS.

$$
578 \div 2=289
$$

Student has a correct answer and shows complete procedure to solve the problem ( $\$ 772.50$ and \$194.50, finds the difference, and divides by 2). Student also explains why these steps were done.

## STUDENT RESPONSE

## RESPONSE SCORE: 3 POINTS

11. Albert sells baseball programs at a stadium. The function $m(x)=2.50 x$ represents the total amount of money collected, in dollars, for selling $x$ baseball programs.
A. Fill in the table with the amounts of money collected for selling baseball programs.

Albert's Revenue

| Baseball <br> Programs <br> Sold | Money <br> Collected (\$) |
| :---: | :---: |
| 150 | $\$ 375$ |
| 175 | $\$ 437.5$ |
| 197 | $\$ 492.5$ |

Student has correctly filled in the table.

The cost, in dollars, to print up $x$ programs for each game is represented by the function $c(x)=0.50 x+40$.
B. On the grid below, draw a line that contains the coordinate points of the cost to print up $x$ programs for each game.


Student has correctly drawn the line on the grid.

Go to the next page to finish question 11.
11. Continued. Please refer to the previous page for task explanation.

In addition to his hourly wage, Albert earns a bonus when the amount of money collected is greater than the cost to print the total number of programs he sold. His bonus is equal to $\frac{1}{2}$ of the difference between the amount of money collected, $m(x)=2.50 x$, and the cost, $c(x)=0.5 x+40$.
C. How much money does Albert earn as a bonus when he sells 309 baseball programs? Show all of your work. Explain why you did each step.

$$
\begin{aligned}
& \quad 309(2.50)=\$ 772.5 \\
& \text { (.5) } 309+40=\$ 194.5 \\
& \text { albert earned } \$ 289 \\
& \text { as a bonus. }
\end{aligned}
$$

Student has a correct answer. The support provided is incomplete; the procedure to solve the problem is complete ( $\$ 772.50$ and $\$ 194.50$, finds the difference, and divides by 2 ), but there is no explanation of why the steps were done.

## STUDENT RESPONSE

## RESPONSE SCORE: 2 POINTS




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## STUDENT RESPONSE

## RESPONSE SCORE: 1 POINT

11. Albert sells baseball programs at a stadium. The function $m(x)=2.50 x$ represents the total amount of money collected, in dollars, for selling $x$ baseball programs.
A. Fill in the table with the amounts of money collected for selling baseball programs.

Albert's Revenue

| Baseball <br> Programs <br> Sold | Money <br> Collected (\$) |
| :---: | :---: |
| 150 | 375 |
| 175 | 437.5 |
| 197 | 492.5 |

Student has correctly filled in the table.

The cost, in dollars, to print up $x$ programs for each game is represented by the function $c(x)=0.50 x+40$.
B. On the grid below, draw a line that contains the coordinate points of the cost to print up $x$ programs for each game.


Student has partial credit for a correct $y$-intercept.

Go to the next page to finish question 11.
11. Continued. Please refer to the previous page for task explanation.

In addition to his hourly wage, Albert earns a bonus when the amount of money collected is greater than the cost to print the total number of programs he sold. His bonus is equal to $\frac{1}{2}$ of the difference between the amount of money collected, $m(x)=2.50 x$, and the cost, $c(x)=0.5 x+40$.
C. How much money does Albert earn as a bonus when he sells 309 baseball programs? Show all of your work. Explain why you did each step.

$$
\begin{aligned}
& 2.5(309)=772.5 \\
& 194.4
\end{aligned}
$$



Student has an incorrect answer. The support provided is insufficient for credit, and there is no explanation of why the steps were done.

## STUDENT RESPONSE

## RESPONSE SCORE: 0 POINTS



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12. A large bucket that is full of water has a small leak on the bottom. The bucket loses water at the rate of 0.5 gallon per minute. After 6 minutes the bucket contains exactly 9 gallons of water.
A. How many gallons of water were initially in the bucket?

## gallons

B. Write an equation in point-slope form to model the number of gallons $(y)$ of water in the bucket after $x$ minutes.
point-slope equation: $\qquad$

Go to the next page to finish question 12.
12. Continued. Please refer to the previous page for task explanation.
C. How many minutes does it take for the bucket to lose 7.5 gallons of water?
$\qquad$ minutes
D. What is the total number of minutes it will take for the bucket to be completely empty?
$\qquad$ minutes

## SCORING GUIDE

## \#12 ITEM INFORMATION

| Alignment | A1.2.2 | Depth of Knowledge | 2 | Mean Score | 1.94 |
| :--- | :--- | :--- | :--- | :--- | :--- |

## ITEM-SPECIFIC SCORING GUIDELINE

| Score | Description |
| :---: | :--- |
| $\mathbf{4}$ | The student demonstrates a thorough understanding of applying the concept of linear rate <br> of change (slope) to solve problems, writing a linear equation in point-slope form when given <br> the slope and 2 points on the line, and determining the $y$-intercept represented by a linear <br> equation by correctly solving problems and clearly explaining procedures. |
| $\mathbf{3}$ | The student demonstrates a general understanding of applying the concept of linear rate of <br> change (slope) to solve problems, writing a linear equation in point-slope form when given <br> the slope and 2 points on the line, and determining the $y$-intercept represented by a linear <br> equation by correctly solving problems and clearly explaining procedures with only minor <br> errors or omissions. |
| $\mathbf{2}$ | The student demonstrates a partial understanding of applying the concept of linear rate of <br> change (slope) to solve problems, writing a linear equation in point-slope form when given <br> the slope and 2 points on the line, and determining the $y$-intercept represented by a linear <br> equation by correctly performing a significant portion of the required task. |
| $\mathbf{1}$ | The student demonstrates minimal understanding of applying the concept of linear rate of <br> change (slope) to solve problems, writing a linear equation in point-slope form when given <br> the slope and 2 points on the line, and determining the $y$-intercept represented by a linear <br> equation. |
| $\mathbf{0}$ | The student does not demonstrate any understanding of the mathematical concepts and <br> procedures as required by the task. Response may show only information copied from the <br> question. |

## Top Scoring Response:

| Part A Answer |  |
| :--- | :--- |
| What? |  |
| 12 |  |

(1 score point)
1 point for correct answer

| Part B Answer |  |
| :--- | :--- |
| What? |  |
| $(y-9)=-0.5(x-6)$ |  |
| OR equivalent equation in <br> point-slope form |  |

(1 score point)
1 point for correct answer

| Part C Answer |  |
| :--- | :--- |
| What? |  |
| 15 |  |

(1 score point)
1 point for correct answer

| Part D Answer |  |
| :--- | :--- |
| What? |  |
| 24 |  |

(1 score point)
1 point for correct answer

## STUDENT RESPONSE

RESPONSE SCORE: 4 POINTS



## STUDENT RESPONSE

## RESPONSE SCORE: 3 POINTS




## STUDENT RESPONSE

RESPONSE SCORE: 2 POINTS



## STUDENT RESPONSE

## RESPONSE SCORE: 1 POINT

12. A large bucket that is full of water has a small leak on the bottom. The bucket loses water at the rate of 0.5 gallon per minute. After 6 minutes the bucket contains exactly 9 gallons of water.
A. How many gallons of water were initially in the bucket?

B. Write an equation in point-slope form to model the number of gallons $(y)$ of water in the bucket after $x$ minutes.

Student has given an incorrect equation.
point-slope equation: $y=0.5 x-9$

Go to the next page to finish question 12.
12. Continued. Please refer to the previous page for task explanation.
C. How many minutes does it take for the bucket to lose 7.5 gallons of water?
3.75

Student has given an incorrect answer. minutes
D. What is the total number of minutes it will take for the bucket to be completely empty?
$12 \cdot 0.5=6$
$\qquad$ minutes

## STUDENT RESPONSE

## RESPONSE SCORE: 0 POINTS

12. A large bucket that is full of water has a small leak on the bottom. The bucket loses water at the rate of 0.5 gallon per minute. After 6 minutes the bucket contains exactly 9 gallons of water.
A. How many gallons of water were initially in the bucket?
B. Write an equation in point-slope form to model the number of gallons $(y)$ of water in the bucket after $x$ minutes.

Student has given an incorrect equation.
point-slope equation: $y=6 x+0.5$
Go to the next page to finish question 12.
12. Continued. Please refer to the previous page for task explanation.
C. How many minutes does it take for the bucket to lose 7.5 gallons of water?

Student has given an incorrect answer.
minutes
D. What is the total number of minutes it will take for the bucket to be completely empty?

## ALGEBRA I MODULE 2-SUMMARY DATA

## MULTIPLE-CHOICE

| Sample <br> Number | Alignment | Answer <br> Key | Depth of <br> Knowledge | $\boldsymbol{p}$-values |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A |  | C | D |  |  |  |  |
| 1 | A1.2.1.1.2 | A | 2 | $38 \%$ | $14 \%$ | $15 \%$ | $32 \%$ |  |
| 2 | A1.2.1.2.1 | C | 2 | $10 \%$ | $18 \%$ | $65 \%$ | $6 \%$ |  |
| 3 | A1.2.1.2.2 | C | 2 | $8 \%$ | $6 \%$ | $80 \%$ | $5 \%$ |  |
| 4 | A1.2.2.1.1 | A | 2 | $83 \%$ | $8 \%$ | $5 \%$ | $4 \%$ |  |
| 5 | A1.2.2.1.4 | A | 2 | $53 \%$ | $2 \%$ | $11 \%$ | $34 \%$ |  |
| 6 | A1.2.2.2.1 | B | 2 | $19 \%$ | $61 \%$ | $11 \%$ | $9 \%$ |  |
| 7 | A1.2.3.1.1 | D | 2 | $11 \%$ | $10 \%$ | $31 \%$ | $48 \%$ |  |
| 8 | A1.2.3.2.1 | B | 2 | $12 \%$ | $37 \%$ | $37 \%$ | $14 \%$ |  |
| 9 | A1.2.3.2.2 | D | 2 | $20 \%$ | $8 \%$ | $14 \%$ | $58 \%$ |  |
| 10 | A1.2.3.2.3 | C | 2 | $5 \%$ | $16 \%$ | $72 \%$ | $7 \%$ |  |

## CONSTRUCTED-RESPONSE

| Sample <br> Number | Alignment | Points | Depth of <br> Knowledge | Mean Score |
| :---: | :---: | :---: | :---: | :---: |
| 11 | A1.2.1 | 4 | 3 | 1.62 |
| 12 | A1.2.2 | 4 | 2 | 1.94 |

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## KEYSTONE EXAMS ALGEBRA I

## ITEM AND SCORING SAMPLER 2014

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[^0]:    ${ }^{1}$ The permission to copy and/or use these materials does not extend to commercial purposes.

[^1]:    ${ }^{2}$ All $p$-value percentages listed in the item information tables have been rounded.

